The basic point of the model in chapter 1 is to illustrate the following general steps in constructing a model in environmental economics:

- Construct a model of the economy with an environmental externality
- Characterize the Pareto optimum allocation in the economy
- Characterize the market (decentralized) outcome and contrast it with the efficient outcome
- Design an intervention to make the market outcome match the efficient outcome.

Model

- 2 individuals receive utility from 2 private goods X and Z, and disutility from emissions E
- Utility functions are $U_1(x_1,E)+z_1$ and $U_2(x_2,E)+z_2$ for individuals 1 and 2, respectively
- A single firm produces X using emissions according to the production function $X=f(E)$, where $df(E)/dE>0$
- The economy is constrained so that $X=x_1+x_2$ and $Z=z_1+z_2$. Note that the later constraint implies $z_2=Z-z_1$

Pareto problem

The Pareto problem is to maximize utility for person 1 subject to a reference utility for person 2 and the technology and scarcity constraints:

$$\max_{x_1,x_2,z_1,E} L = U_1(x_1,E)+z_1+\lambda\left(\bar{u}_1 - U_2(x_1,E) - \underbrace{(Z-z_1)}_{z_2}\right)+\gamma\left(f(E)-x_1-x_1\right)$$

First order conditions:

$$x_1: \quad \frac{\partial U_1(\cdot)}{\partial x_1}=\gamma$$

$$x_2: \quad -\lambda\frac{\partial U_2(\cdot)}{\partial x_2}=\gamma$$

$$z_1: \quad \lambda=-1$$

$$E: \quad \frac{\partial U_1(\cdot)}{\partial E}-\lambda\frac{\partial U_2(\cdot)}{\partial E}=-\gamma f'(E)$$

Characterization of the Pareto optimum:

$$\frac{\partial U_1(\cdot)}{\partial x_1}=\frac{\partial U_2(\cdot)}{\partial x_2}=\gamma$$

$$-\frac{\frac{\partial U_1(\cdot)}{\partial E}}{\frac{\partial U_1(\cdot)}{\partial x_1}}-\frac{\frac{\partial U_2(\cdot)}{\partial E}}{\frac{\partial U_2(\cdot)}{\partial x_2}}=f'(E)$$

Market problem

Individuals maximize utility, taking p (the competitive price of X), incomes y_1 and y_2, and E as given

$$\max_{x_1} U_1(x_1, E) + (y_1 - px_1)$$

$$\max_{x_2} U_2(x_2, E) + (y_2 - px_2)$$

Individuals' first order conditions:

$$x_1: \quad \frac{\partial U_1(\cdot)}{\partial x_1} = p$$

$$x_2: \quad \frac{\partial U_2(\cdot)}{\partial x_2} = p$$

Firm maximizes profit, taking p and technology as given

$$\max_E pf(E)$$

Firm's first order condition:

$$E: \quad pf'(E) = 0$$

Characterization of market solution:

$$\frac{\partial U_1(\cdot)}{\partial x_1} = \frac{\partial U_2(\cdot)}{\partial x_2}$$

$$f'(E) = 0$$

Note that the Pareto conditions in blue do not match the market conditions in red – there is a market failure due to the externality in pollution.

Environmental policy

Consider setting a tax τ per unit of emissions that the polluting firm must pay. Under this policy the firm's objective function is

$$\max_E pf(E) - \tau E$$

New first order condition:

$$E: \quad pf'(E) = \tau$$

Possible value for τ

$$\tau = p\left[-\frac{\frac{\partial U_1(\cdot)}{\partial E}}{\frac{\partial U_1(\cdot)}{\partial x_1}} - \frac{\frac{\partial U_2(\cdot)}{\partial E}}{\frac{\partial U_2(\cdot)}{\partial x_2}}\right]$$

With this value for the environmental tax the first order profit and utility maximization conditions imply:

$$\left[-\frac{\frac{\partial U_1(\cdot)}{\partial E}}{\frac{\partial U_1(\cdot)}{\partial x_1}}-\frac{\frac{\partial U_2(\cdot)}{\partial E}}{\frac{\partial U_2(\cdot)}{\partial x_2}}\right]=f'(E), \qquad \frac{\partial U_1(\cdot)}{\partial x_1}=\frac{\partial U_2(\cdot)}{\partial x_2}$$

Note that the decentralized outcome under environmental policy in green matches the Pareto characterization in blue.